2. Schularbeit

(B) 1) $\nu: y = x^3$, $P(2|y_P) \Rightarrow P(2|8)$

$Q(1|y_Q) \Rightarrow Q(1|1)$

a) $g_{PQ}: \overrightarrow{QP} = \begin{pmatrix}1\\7\end{pmatrix} \perp \begin{pmatrix}7\\-1\end{pmatrix} \Rightarrow g_{PQ}: 7x - y = 6$

$-y = 6 - 7x$
$y = -6 + 7x$

$g_{PQ} \cap \nu = \{P, Q \text{ (sic!)}, R\}$:

$$-6 + 7x = x^3$$
$$0 = \underbrace{x^3 - 7x + 6}_{a(x)}$$

	1	0	−7	6
2	1	2	−3	0
1	1	3	0	

$\Rightarrow a(x) = (x-2)\cdot(x-1)\cdot(x+3)$

$\Rightarrow x_1 = 2$ ($\hat{=}$ P)
$x_2 = 1$ ($\hat{=}$ Q)
$x_3 = -3 \Rightarrow R(-3|-27)$

b) $t_T: y = f(x_T) + f'(x_T)\cdot(x - x_T)$

$y' = 3x^2$

$y'(2) = 12 \Rightarrow t_P: y = 8 + 12(x-2)$

$y'(1) = 3 \Rightarrow t_Q: y = 1 + 3(x-1)$

$y'(-3) = +27 \Rightarrow t_R: y = -27 + 27(x+3)$

c) $t_P \cap \nu = \{P, P'\}$: $8 + 12(x-2) = x^3$

$$8 + 12x - 24 = x^3$$
$$0 = \underbrace{x^3 - 12x + 16}_{b(x)}$$

	1	0	−12	16
2	1	2	−8	0
2	1	4	0	

$\Rightarrow b(x) = (x-2)^2\cdot(x+4) \Rightarrow x_1 = x_2 = 2$ ($\hat{=}$ P)
$x_3 = -4$

$\Rightarrow \underline{P'(-4|-64)}$

$t_Q \cap \gamma = \{Q, Q'\}$:
$$1+3(x-1) = x^3$$
$$1+3x-3 = x^3$$
$$0 = x^3 - 3x + 2$$
$c(x)$

	1	0	-3	2
1	1	1	-2	0
1	1	2	0	

$\Rightarrow c(x) = (x-1)^2 \cdot (x+2)$

$\Rightarrow x_1 = x_2 = 1 \ (\hat{=} Q)$

$\underline{x_3 = -2}$

$\Rightarrow \underline{Q'(-2|-8)}$

$t_R \cap \gamma = \{R, R'\}$:
$$-27 + 27(x+3) = x^3$$
$$-27 + 27x + 81 = x^3$$
$$0 = x^3 - 27x - 54$$
$d(x)$

	1	0	-27	-54
-3	1	-3	-18	0
-3	1	-6	0	

$\Rightarrow d(x) = (x+3)^2 \cdot (x-6)$

$\Rightarrow x_1 = x_2 = -3 \ (\hat{=} R)$

$\underline{x_3 = 6}$

$\Rightarrow \underline{R'(6|216)}$

$$\left.\begin{array}{l} \overrightarrow{P'Q'} = \begin{pmatrix} 2 \\ 56 \end{pmatrix} = 2 \cdot \begin{pmatrix} 1 \\ 28 \end{pmatrix} \\ \overrightarrow{Q'R'} = \begin{pmatrix} 8 \\ 224 \end{pmatrix} = 8 \cdot \begin{pmatrix} 1 \\ 28 \end{pmatrix} \end{array}\right\} \Rightarrow \overrightarrow{P'Q'} \parallel \overrightarrow{Q'R'}$$
, d.h. P', Q', R' liegen kollinear

Vektoren!